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**COMPARATIVE STUDY BETWEEN LIQUID STORAGE TANKS WITH SLIDING**  
**ISOLATORS HAVING VARIABLE CURVATURE (SIVC)**

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**ABSTRACT**

This is a comparative study where the seismic response of a ground supported liquid storage tank is analyzed incorporating four different isolators belonging to the same category of sliding isolator having variable curvature (SIVC). In recent years, various isolators have been developed with numerous different properties. One of them is curvature, a property by which time-period of structure is highly influenced. Here, a comparative study is executed between four isolators namely Variable curvature friction pendulum isolator of the order 4<sup>th</sup> (VCFP O4), Variable curvature friction pendulum isolator of the order 6<sup>th</sup> (VCFP O6), Polynomial defined sliding isolator having variable curvature of the order 3<sup>rd</sup> (PSIVC-3<sup>rd</sup>) and Polynomial defined sliding isolator having variable curvature of the order 5<sup>th</sup> (PSIVC-5<sup>th</sup>) which have variable curvature influenced by their polynomial functions. The effectiveness of these four passive base isolation systems for vibration control of ground supported liquid storage tank under near-fault ground excitations has been investigated. Newmark's linear acceleration method is utilized in this study for solving equations governing the motion of the structure. From this comparative study, VCFP O4 is found to be most effective for both slender as well as broad tanks in reducing the seismic response.

**Keywords:** *Sliding isolators having variable curvature (SIVC); Variable curvature friction pendulum (VCFP); Polynomial sliding isolator having variable curvature (PSIVC); near-fault ground motion.*

**I. INTRODUCTION**

Earthquake is the disaster where, to a very high extent loss of lives and damage to the structure is encountered. Various techniques have been developed that reduce the impact experienced by a structure during an earthquake to a considerable degree. One of the most effective techniques is base isolation. Furthermore, by introducing variation in the curvature, the possibility of resonance that was encountered in isolators like FPS would be ruled out.

Ground supported liquid storage tanks are used for storing various types of liquids depending on factors such as viscosity, flammability etc. These storage tanks are used in industries, nuclear/thermal power plants, irrigation/agriculture and domestic purposes. The failure of these structures would result into utility loss, economic loss as well as environmental pollution due to spillage of liquids that are not suitable for direct interaction with the environment. Moreover, tanks containing flammable liquids can even lead to extensive fire in the surroundings which would also ultimately lead to loss of life.

In the past years, base isolation has emerged as a significant vibration control system. Numerous upgrades have also been witnessed resulting into even better efficiency. These systems lengthen the fundamental time period of structural vibration against the period of the ground excitations during an earthquake. Instead of increasing the resistance, the isolators mitigate the impact of vibrations by decoupling the structure decreasing the seismic demand on the structure.

Isolators having variable curvature such as, Variable curvature friction pendulum isolator of the order 4<sup>th</sup> (VCFP O4), Variable curvature friction pendulum isolator of the order 6<sup>th</sup> (VCFP O6), Polynomial defined sliding isolator having variable curvature of the order 3<sup>rd</sup> (PSIVC-3<sup>rd</sup>) and Polynomial defined sliding isolator having variable curvature 5<sup>th</sup> order (PSIVC-5<sup>th</sup>) have been compared on the basis of their effectiveness for vibration control of liquid

storage tanks under two near-fault ground excitations as recorded in Table 1.

*Table 1 Description of near-field ground excitations considered in this investigation*

Near-Fault ground excitations	Normal component		
	PGD (cm)	PGV (cm/s)	PGA (g)
1979, Imperial Valley (El Centro Array #5)	76.5	98	0.37
1992, Landers (Lucerne Valley)	230	136	0.71

## II. GEOMETRIC FUNCTIONS OF ISOLATORS

*a. General explanation and Common mathematical preliminary of all the four isolators used in this study:*

All the four isolators used in this study consist of a sliding concave surface which is fixed, with no relative displacement to the ground and a slider connected the super-structure for decoupling the super-structure from the motion of the ground. Here, as the curvature varies, the restoring force and isolation frequency also change with the displacement of isolator.

The total shear  $F_s(x)$  for isolators in the sliding state [1-2] can be represented by

$$F_s(x) = F_f(x) + F_r(x) \quad (1)$$

Simplifying,

$$F_f(x) \approx \mu W sgn(\dot{x}) \quad (2)$$

$$F_r(x) = W y'(x) \quad (3)$$

where  $F_f(x)$  and  $F_r(x)$  indicate the friction force and restoring force respectively. The total weight of super-structure is denoted by  $W$ .  $y'(x)$  is derivative of the first order of the geometric function  $y(x)$ .  $\mu$  denotes coefficient of friction, assumed to be constant throughout this study.

The isolator stiffness  $k_r(x)$  of the considered isolators in the study, can be explained as rate of change of the restoring force  $F_r(x)$  with respect to displacement of the slider, i.e.,

$$k_r(x) = W y''(x) \quad (4)$$

And  $\omega(x)$  which represents the tangential isolation frequency & can be enumerated by

$$\omega(x) = \sqrt{g y''(x)} \quad (5)$$

where  $g$  denotes gravitational acceleration.

b. Polynomial defined Sliding Isolator having Variable Curvature (PSIVC 3<sup>rd</sup> order):

From the geometric function of PSIVC 3<sup>rd</sup> order [1], the restoring force can also be explicated by the subsequent polynomial function of third-order:

$$y'(x) = \frac{F_r(x)}{P} = ax^3 + cx \tag{6}$$

Thus, the normalized isolator stiffness with reference to the vertical load  $P$  would be,

$$y''(x) = \frac{k_r(x)}{P} = 3ax^2 + c \tag{7}$$

where  $a$  and  $c$  are constant coefficients. The polynomial coefficients are replaced by three design parameters  $k_0$ ,  $k_1$  and  $D$ . The relation between design parameters and polynomial coefficients are as follows:

$$a = \frac{(k_1 - k_0)}{3(D)^2} \tag{8}$$

$$c = k_0 \tag{9}$$

where  $k_0$  indicates initial stiffness at the displacement  $x = 0$ , while  $k_1$  indicates tangential stiffness at the displacement  $x = D$ , i.e.,  $k_1(x = D)$ .

c. Polynomial defined Sliding Isolator having Variable Curvature (PSIVC 5<sup>th</sup> order):

From the geometric function of PSIVC 5<sup>th</sup> order [1], the restoring force can be explicated by the subsequent polynomial function of fifth-order:

$$y'(x) = \frac{F_r(x)}{P} = ax^5 + cx^3 + ex \tag{10}$$

Thus, the normalized isolator stiffness with reference to the vertical load  $P$  would be,

$$y''(x) = \frac{k_r(x)}{P} = 5ax^4 + 3cx^2 + e \tag{11}$$

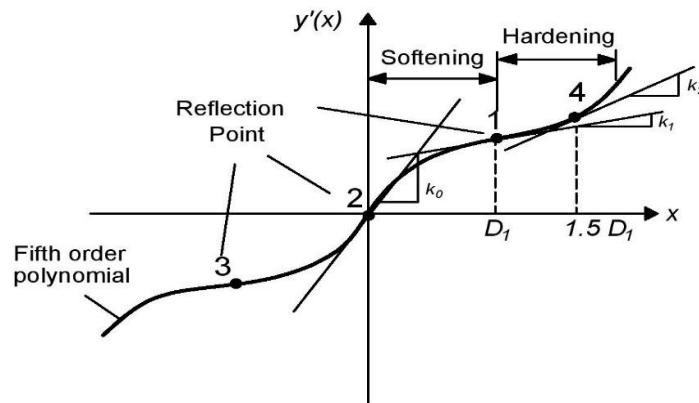


Figure 1 Variation of normalized restoring force  $y'(x)$  for fifth order polynomial function [1]

where three constant coefficients  $a$ ,  $c$  and  $e$  are replaced by three design parameters  $k_0$ ,  $k_1$  and  $D$ . The relation between design parameters and polynomial coefficients are as follows:

$$a = \frac{(-k_0 + k_1)}{-5(D)^4} \tag{12}$$

$$c = \frac{2(-k_0 + k_1)}{3(D)^2} \tag{13}$$

$$e = k_0 \tag{14}$$

where  $k_0$  indicates initial stiffness at the displacement  $x = 0$ , while  $k_1$  indicates tangential stiffness at the displacement  $x = D$ , i.e.,  $k_1 (x = D)$ .

d. Variable Curvature Friction Pendulum Isolator (VCFP O4):

From the geometric function of VCFP O4[2], the restoring force can be explicated by the subsequent polynomial function of third-order:

$$y'(x) = \frac{F_r(x)}{P} = 4p_1x^3 + 2p_2x \tag{15}$$

Thus, the normalized isolator stiffness with reference to the vertical load P would be,

$$y''(x) = \frac{k_r(x)}{P} = 12p_1x^2 + 2p_2 \tag{16}$$

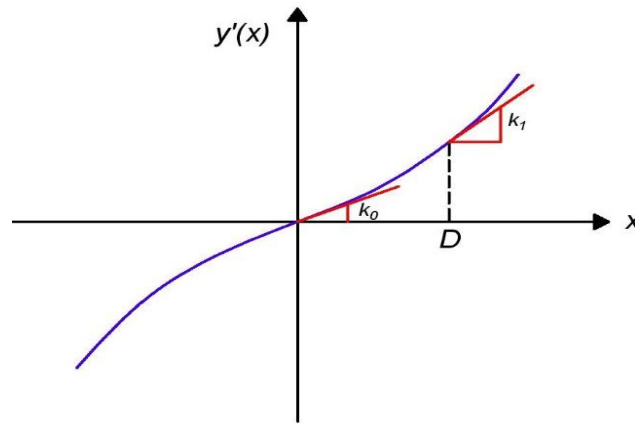


Figure 2 Variation of normalized restoring force  $y'(x)$  for fourth order polynomial function [2]

where  $D$  is the specific displacement with related stiffness  $k_1$  and the relation between their coefficients and engineering parameters are as follows:

$$p_1 = \frac{(k_1 - k_0)}{12(D)^2} \tag{17}$$

$$p_2 = \frac{(k_0)}{2} \tag{18}$$

where  $k_0$  is the normalized initial stiffness at the displacement  $x = 0$  and is further simplified as,

$$k_0 = \left(\frac{2\pi}{T_0}\right)^2 / g \tag{19}$$

where  $g$  denotes gravitational acceleration and  $T_0$  indicates initial time period.

e. Variable Curvature Friction Pendulum Isolator (VCFP O6)-

From the geometric function of VCFP O6[2], the restoring force can be explicated by the subsequent polynomial of fifth-order:

$$y'(x) = \frac{F_r(x)}{P} = 6q_1x^5 + 4q_2x^3 + 2q_3x \tag{20}$$

Thus, the normalized isolator stiffness with reference to the vertical load  $P$  would be,

$$y''(x) = \frac{k_r(x)}{P} = 30q_1x^4 + 12q_2x^2 + 2q_3 \tag{21}$$

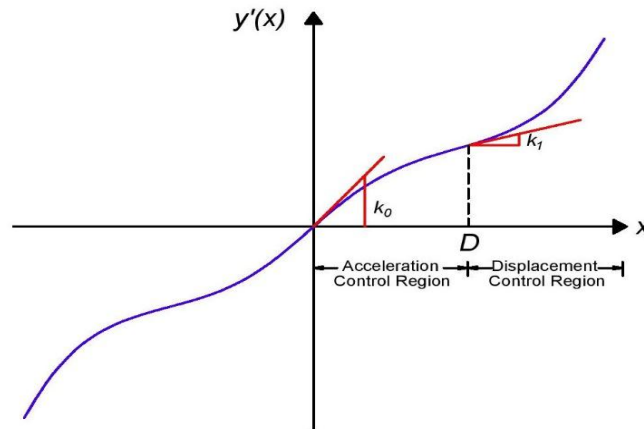


Figure 3 Variation of normalized restoring force  $y'(x)$  for sixth order polynomial function [2]

where  $D$  is the specific displacement with related stiffness  $k_1$  and the relation between their coefficients and engineering parameters are as follows:

$$q_1 = \frac{(k_0 - k_1)}{30(D)^4} \tag{22}$$

$$q_2 = \frac{(k_1 - k_0)}{12(D)^2} \tag{23}$$

$$q_3 = \frac{(k_0)}{2} \tag{24}$$

where  $k_0$  is the normalized initial stiffness at the displacement  $x = 0$  and is further simplified as,

$$k_0 = \left(\frac{2\pi}{T_0}\right)^2 / g \tag{25}$$

where  $g$  denotes gravitational acceleration and  $T_0$  indicates initial time period.

### III. STRUCTURAL MODELLING

Steel liquid storage tank is used for base isolation modeling suggested by Haroun [3]. Figure 4 shows liquid storage tank isolated with above mentioned isolators one at a time. The contained liquid stored in tank is assumed as an inviscid, incompressible and has irrotational flow. At the time of earthquake, whole liquid mass of tank vibrates in three patterns such as convective or sloshing, impulsive and rigid mass. A convective and impulsive mass vibrates in different modes but first convective mode is considered and then the impulsive mode is considered for response assumption. Here, the Impulsive mass ( $m_i$ ), Sloshing mass ( $m_c$ ) and rigid mass ( $m_r$ ) are called lumped masses. Three degree of freedom system is considered for analysis, i.e.,  $u_b$ ,  $u_i$  and  $u_c$ , which indicates the absolute rigid, impulsive and sloshing displacement of masses, respectively.

The different assumptions which are used in the system are:

1. The tanks self-weight is neglected due to the fact that it's very small.
2. Damping ratio is assumed for damping coefficient of the motion of impulsive and convective masses.
3. The coefficient of friction of the following isolators does not depend upon the relative velocity of sliding surfaces.
4. It is assumed that the slider of the isolator remains in contact with the sliding surfaces.

Mathematical modeling of ground rested liquid storage tank isolated by above discussed isolators is shown below:

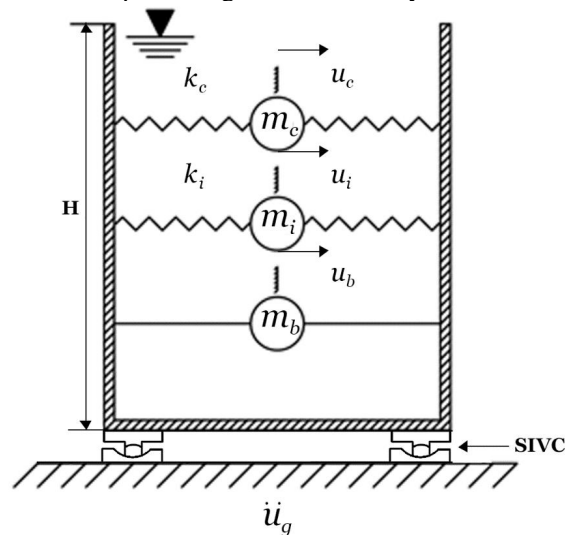


Figure 4 Mathematical modelling of liquid storage tank isolated by SIVC

The  $m_c$ ,  $m_i$  and  $m_r$  in relation to liquid mass,  $m$  and various mass ratios for  $t_b/R = 0.004$  are introduced as:

$$m_c = Y_c m \quad (27)$$

$$m_i = Y_i m \quad (28)$$

$$m_r = Y_r m \quad (29)$$

$$m = \pi R^2 H \rho_w \quad (30)$$

$$Y_c = 1.01327 - 0.87578 * S + 0.35708 * S^2 - 0.06692 * S^3 + 0.00439 * S^4 \quad (31)$$

$$Y_i = -0.15467 + 1.21716 * S - 0.62839 * S^2 + 0.14434 * S^3 - 0.0125 * S^4 \quad (32)$$

$$Y_r = -0.01599 + 0.86356 * S - 0.30941 * S^2 + 0.04083 * S^3 \quad (33)$$

where  $Y_c$ ,  $Y_i$  and  $Y_r$  are the ratios of the masses mentioned above; aspect ratio of tank,  $S=H/R$ ;  $\rho_w$  denotes mass density of containing liquid;  $R$  is tank radius; and  $H$  is liquid height.

Following equation shows the fundamental frequency of convective and impulsive mass,  $\omega_c$  and  $\omega_i$ , respectively:

$$\omega_c = \sqrt{1.84 \left(\frac{g}{R}\right) \tanh(1.84S)} \quad (34)$$

$$\omega_i = \frac{P}{H} \sqrt{\frac{E}{\rho_s}} \quad (35)$$

where  $g$  is gravitational acceleration;  $E$  denotes elasticity modulus,  $\rho_s$  denotes wall density of tank and  $P$  is given by:

$$P = 0.037085 + 0.084302 * S - 0.05088 * S^2 + 0.012523 * S^3 - 0.0012 * S^4 \quad (36)$$

The damping and stiffness, equivalent with the impulsive and convective masses are introduced as:

$$c_c = 2\xi_c m_c \omega_c \quad (37)$$

$$c_i = 2\xi_i m_i \omega_i \quad (38)$$

$$k_c = m_c \omega_c^2 \quad (39)$$

$$k_i = m_i \omega_i^2 \quad (40)$$

where  $\xi_i$  and  $\xi_c$  denotes the ratio of damping corresponding to impulsive and sloshing mass, respectively.

The equation governing the motion is expressed as given below in matrix form for liquid storage tank with isolation:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} + \{F\} = -[m]\{r\}\ddot{u}_g \quad (41)$$

where  $[m]$  explicates mass matrix,  $[c]$  explicates damping matrix and  $[k]$  explicates stiffness matrix;  $\{x\} = \{x_c, x_i, x_b\}^T$ ;  $x_c = u_c - u_b$  is convective mass displacement,  $x_i = u_i - u_b$  is impulsive mass displacement in relation with bearing displacement and  $x_b = u_b - u_g$  is bearing displacement in relation with the ground motion;  $\{F\} = \{0, 0, F_x\}$  denotes friction force vectors;  $\{r\} = \{0, 0, 1\}^T$  denotes influencing coefficient vector;  $F_x$  denotes friction force in the isolators; and  $\ddot{u}_g$  denotes the seismic ground acceleration.

#### IV. COMPARISON OF RESPONSES

The response of broad as well as slender tank under two different ground excitations are examined. Various essential parameters required to define broad and slender liquid storage tank system are taken from Panchal and Jangid [4]. For broad tank, Aspect ratio,  $S$  is 0.6; Height,  $H$  is 14.6 m; Natural frequencies of the impulsive mass ( $\omega_i$ ) & convective mass ( $\omega_c$ ) are 3.944 Hz & 0.123 Hz, respectively; Elastic modulus,  $E$  is 200 GPa; and Density of mass,  $\rho_s$  is 7900 kg/m<sup>3</sup>. For slender tank, Aspect ratio,  $S$  is 1.85; Height,  $H$  is 11.3 m; Natural frequencies of the impulsive mass ( $\omega_i$ ) & convective mass ( $\omega_c$ ) are 5.963 Hz & 0.273 Hz, respectively; Elastic modulus,  $E$  is 200 GPa; and Density of mass,  $\rho_s$  is 7900 kg/m<sup>3</sup>.

For both the tanks, Thickness of Wall of tank to radius ratio,  $t_w/R$  is 0.004 and Ratio of damping of the sloshing mass ( $\xi_c$ ) & the impulsive mass ( $\xi_i$ ) is 0.5% & 2%, respectively.



Table 2 Design assumptions

Isolator Type	$T_b$ (sec)	$\mu$	$k_0(1/m)$	$k_1(1/m)$	$D$ (m)	$g(m/s^2)$
VCFP O4	2.5	0.06	0.64	4	0.2	9.81
VCFP O6	2.5	0.06	0.64	0	0.2	9.81
PSIVC 3 <sup>rd</sup>	2.5	0.06	0.64	0	0.2	9.81
PSIVC 5 <sup>th</sup>	2.5	0.06	0.64	0	0.2	9.81

Bearing displacement variation of base shear for broad liquid storage tank isolated with VCFP O4, VCFP O6, PSIVC 3<sup>rd</sup> and PSIVC 5<sup>th</sup> for Imperial Valley ground excitation, 1979 (El Centro Array #5) and Landers ground excitation, 1992 (Lucerne Valley) is shown in Figures 5 & 6, respectively.

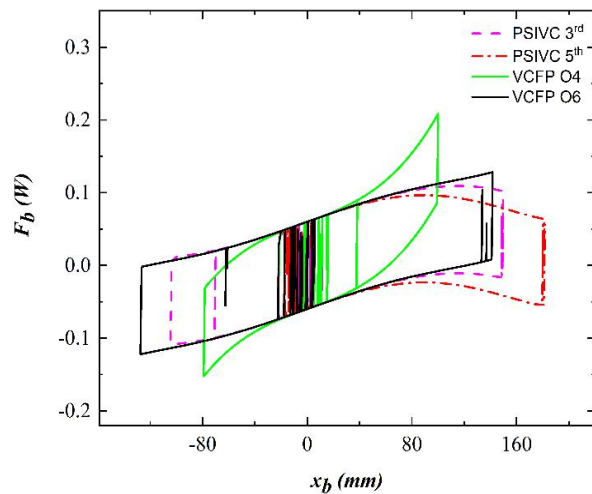


Figure 5 Bearing displacement variation of base shear for broad tank with isolators mentioned above for Imperial Valley ground excitation, 1979 (El Centro Array #5)

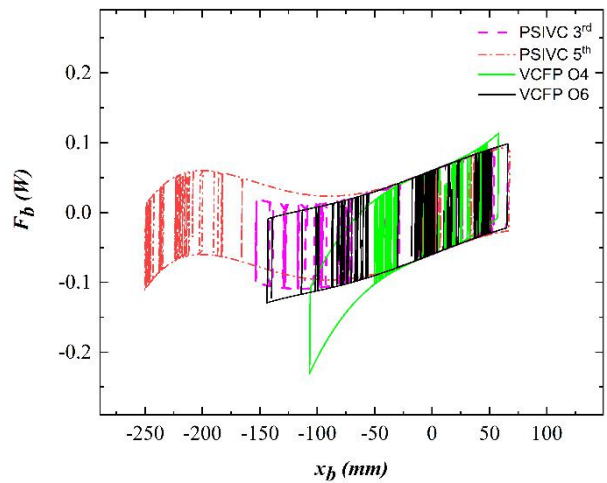


Figure 6 Bearing displacement variation of base shear for broad tank with isolators mentioned above for Landers ground excitation, 1992 (Lucerne Valley)

From the Figures 5&6, it can be noted that under both the earthquakes, PSIVC 5<sup>th</sup> has maximum isolator displacement. Similarly, in terms of base shear it can be concluded that PSIVC 3<sup>rd</sup> attracts least base shear compared to the other three isolators.

Variations with respect to time of base shear, isolator, impulsive and sloshing displacement of broad tank with VCFP O4, VCFP O6, PSIVC 3<sup>rd</sup> and PSIVC 5<sup>th</sup> for Imperial Valley ground excitation, 1979 (El Centro Array #5) & Landers ground excitation, 1992 (Lucerne Valley) is shown in Figures 7 & 8, respectively.



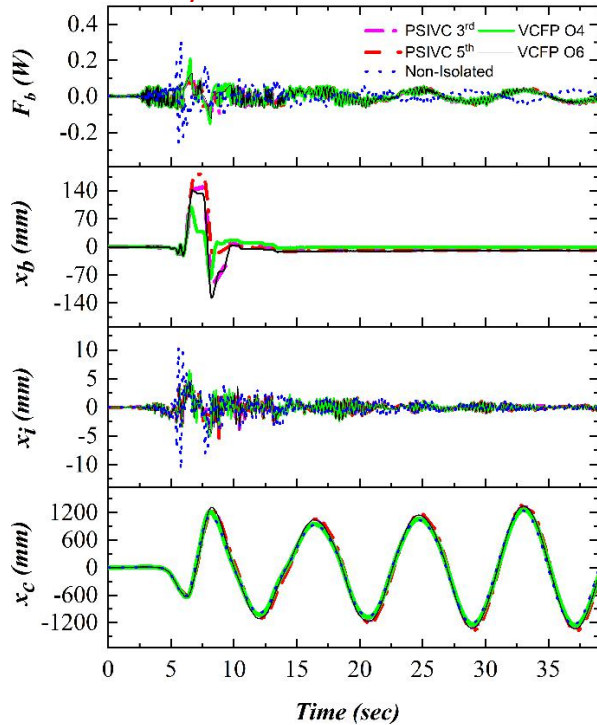


Figure 7 Variations with respect to time of base shear, impulsive, isolator & sloshing displacement of broad tank for Imperial Valley ground excitation, 1979 (El Centro Array #5).

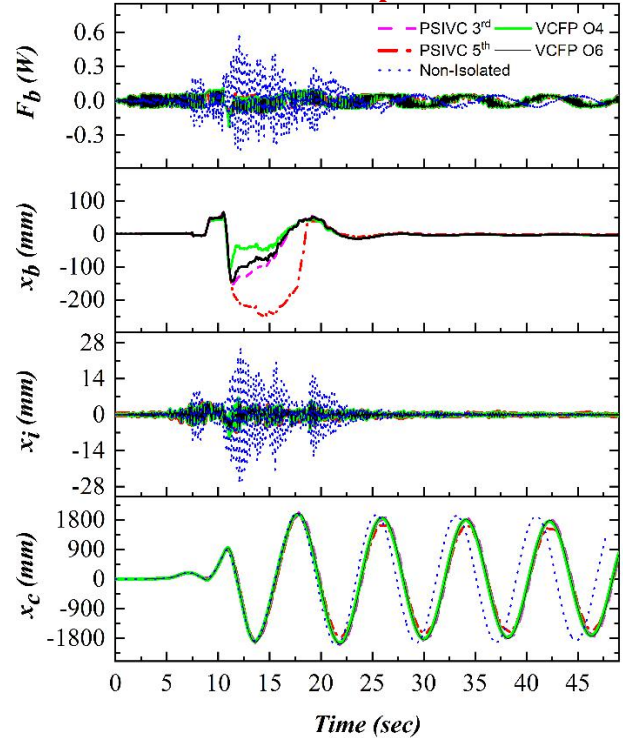


Figure 8 Variations with respect to time of base shear, impulsive, isolator & sloshing displacement of broad tank for Landers ground excitation, 1992 (Lucerne Valley).

Bearing displacement variation of base shear for slender liquid storage tank isolated with VCFP O4, VCFP O6, PSIVC 3<sup>rd</sup> and PSIVC 5<sup>th</sup> for Imperial Valley ground excitation, 1979 (El Centro Array #5) & Landers ground excitation, 1992 (Lucerne Valley) is shown in Figures 9 & 10, respectively.

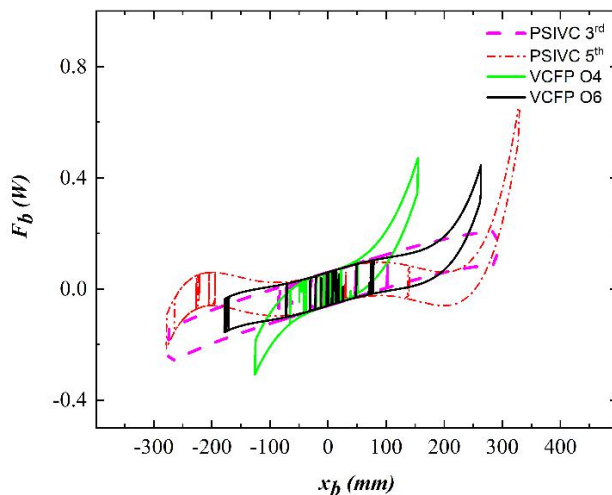


Figure 9 Bearing displacement variation of base shear for slender tank with isolators mentioned above for Imperial Valley ground excitation, 1979 (El Centro Array #5)

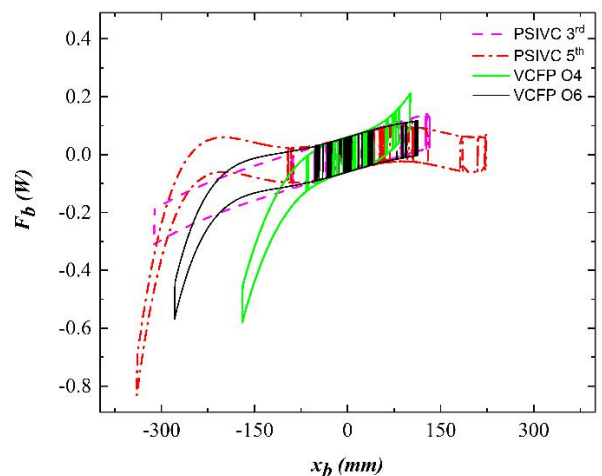


Figure 10 Bearing displacement variation of base shear for slender tank with isolators mentioned above for Landers ground excitation, 1992 (Lucerne Valley)

From Figures 7 & 8, it can be noted that due similarity in the order of the geometric functions, the isolators tend to demonstrate lack of effectiveness and similar kind of behavior, when considering convective displacement  $x_c$ . These figures also indicate that the isolators are fairly effective in controlling  $x_i$  &  $f_b$ .

From the Figures 9 & 10, it can be observed that for both the earthquakes PSIVC 5<sup>th</sup> order attracts maximum isolator displacement and maximum base shear, making it least effective in slender tanks compared to the other three isolators.

Variations with respect to time of base shear, isolator, impulsive and sloshing displacement of slender tank with VCFP O4, VCFP O6, PSIVC 3<sup>rd</sup> and PSIVC 5<sup>th</sup> for Imperial Valley ground excitation, 1979 (El Centro Array #5) & Landers ground excitation, 1992 (Lucerne Valley) is shown in Figures 11 & 12, respectively.

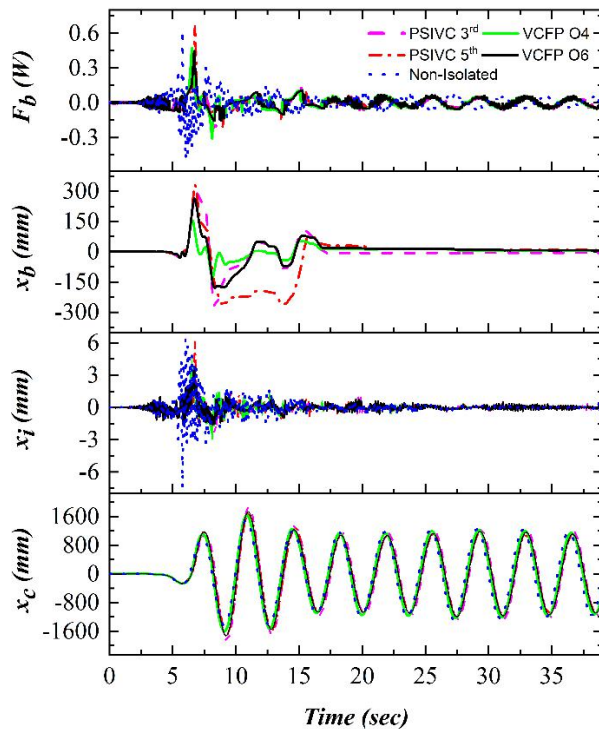


Figure 11 Variations with respect to time of base shear, impulsive, isolator & sloshing displacement of slender tank for Imperial Valley ground excitation, 1979 (El Centro Array #5).

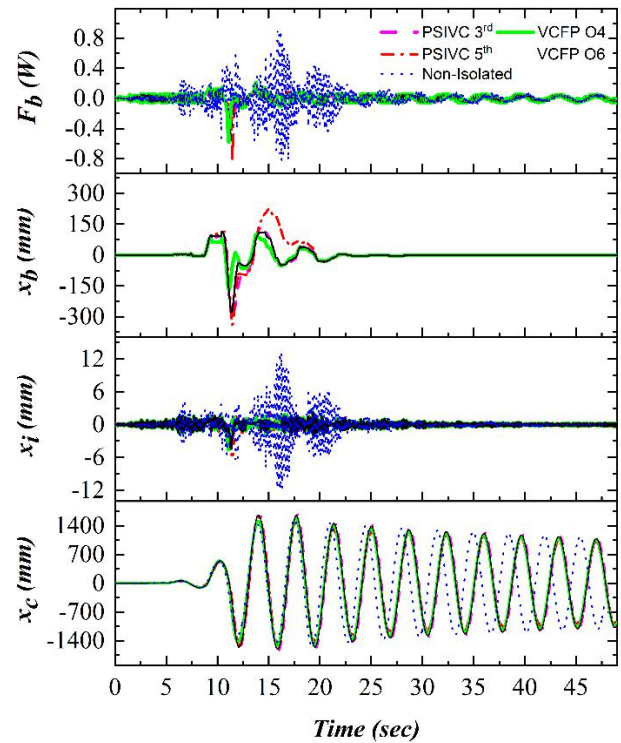


Figure 12 Variations with respect to time of base shear, impulsive, isolator & sloshing displacement of slender tank for Landers ground excitation, 1992 (Lucerne Valley)

From the Figures 11 & 12, it can be observed that for slender liquid storage tanks, the isolators were found to be less effective in terms of controlling, convective displacement  $x_c$ . As noted in broad tanks as well, all the four isolators also portray similar kind of behaviour in terms of convective displacement  $x_c$ .

These figures also indicate that the isolators are fairly effective to control  $f_b$ . Further, all the isolators significantly reduce the impulsive displacement  $x_i$ . Moreover, the base shear  $f_b$  also increases for slender liquid-storage tank compared to broad liquid-storage tank.

Comparison of peak values of different response quantities of both slender and broad tank with VCFP O4, VCFP O6, PSIVC 3<sup>rd</sup> and PSIVC 5<sup>th</sup> are shown in Table 3.

Table 3 Peak values of response quantities of slender & broad liquid storage tank

Earthquake	Isolator	Peak response quantities of slender tank				Peak response quantities of broad tank			
		$F_b (W)$	$x_i (mm)$	$x_c (mm)$	$x_b (mm)$	$F_b (W)$	$x_i (mm)$	$x_c (mm)$	$x_b (mm)$
1979, Imperial Valley (El Centro Array #5)	Non-isolated	0.63	7.58	1511	-	0.31	11.09	1245	-
	PSIVC 3 <sup>rd</sup>	0.26	2.28	1870	290	0.11	4.58	1350	150
	PSIVC 5 <sup>th</sup>	0.67	6.21	1696	329	0.09	5.47	1383	181
	VCFP O4	0.47	4.48	1639	155	0.21	6.43	1258	100
	VCFP O6	0.44	3.8	1728	263	0.13	4.80	1342	142
1992, Landers (Lucerne Valley)	Non-isolated	0.906	13.11	1546	-	0.59	27.02	2016	-
	PSIVC 3 <sup>rd</sup>	0.31	2.53	1690	311	0.11	5.29	2006	153
	PSIVC 5 <sup>th</sup>	0.83	7.05	1602	340	0.11	6.51	1931	250
	VCFP O4	0.57	4.58	1572	170	0.23	8.73	1960	106
	VCFP O6	0.56	4.39	1658	278	0.13	5.68	1998	143

## V. CONCLUSIONS

The base isolated liquid storage tank incorporating four different isolators one at a time namely Variable curvature friction pendulum isolator of the order 4<sup>th</sup>(VCFP O4), Variable curvature friction pendulum isolator of the order 6<sup>th</sup>(VCFP O6), Polynomial defined sliding isolator having variable curvature of the order 3<sup>rd</sup>(PSIVC-3<sup>rd</sup>) and Polynomial defined sliding isolator having variable curvature of the order 5<sup>th</sup> (PSIVC-5<sup>th</sup>) are analyzed to determine the seismic responses under two different ground excitations with the help of Newmark's linear acceleration method.

The conclusions derived from this comparative study are as follows:

- In broad tanks, PSIVC 5<sup>th</sup> is noted to be most effective in terms of reducing base shear and least effective for the same in slender tanks compared to the other three isolators.
- PSIVC 3<sup>rd</sup> exhibits considerable reduction in base shear and impulsive displacement in broad as well as slender tanks compared to the other isolators used in study.
- In terms of isolator displacement, out of the four VCFP O4 exhibits least isolator displacement in both the tanks.

- d. With consideration of all the parameters collectively, namely impulsive displacement, convective displacement, isolator displacement as well as base shear for both broad and slender tanks, on an average VCFP O4 is found to be the most effective.
- e. None of the isolators portray effectiveness in terms of convective displacement.

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